University of Nottingham
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Shaft design

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Learning Objectives

Part 1:

- Introduction to shaft function, types & applications
- Considerations for proper shaft connections

Part 2:

- Methods to evaluate shaft loading and to determine shaft diameter using ASME design code
- Design features to prevent fatigue
- Calculation of shaft deflection & critical speed

Part 3:

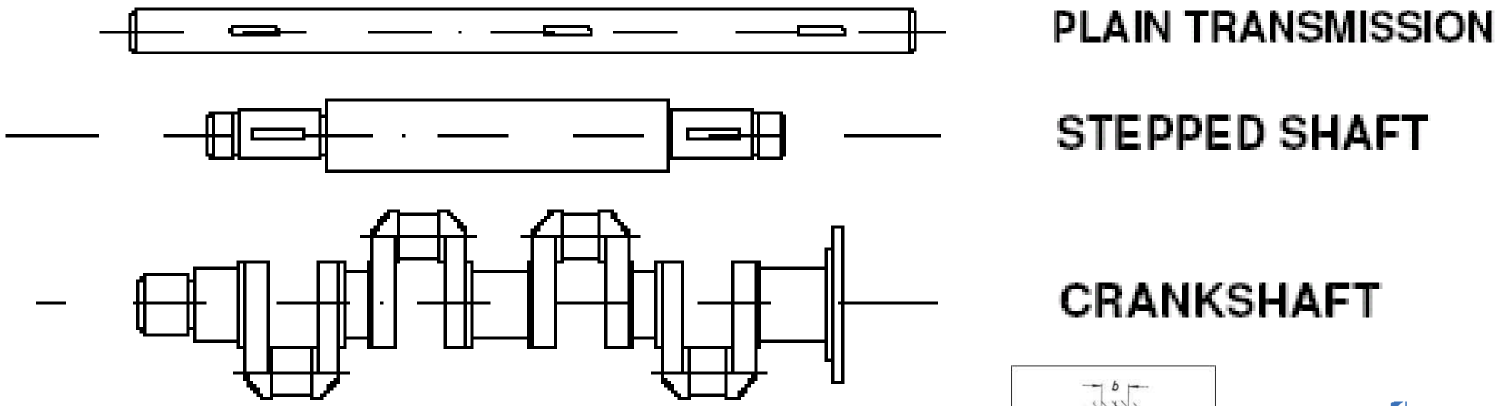
- Worked examples on shaft loading and shaft design calculations

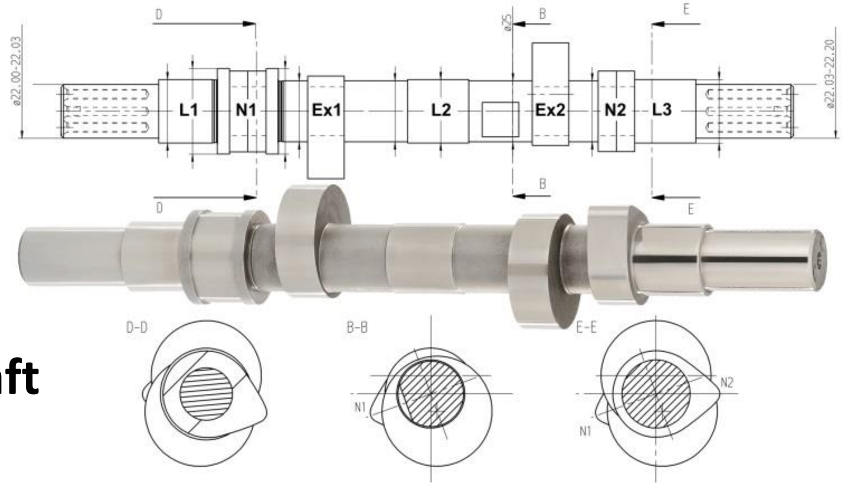
Notes: Effective learning can be achieved in conjunction with Bearings lectures and contents from Statics & Dynamics (MMMME1028) and Mechanics of Solids (MMME2053) on Beam bending and deflection.

Function & Types

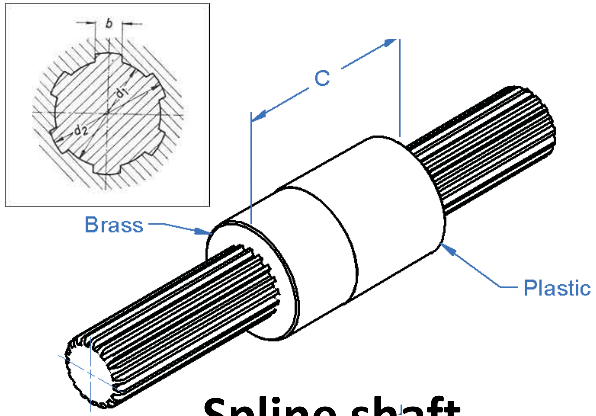
- Function:
  - a **slender component** of circular cross-section that **rotates and transmits power from a driving device**
  - a means to provide **shaft-hub** and **shaft-shaft connections**

- Types:
  - Plain shaft
  - Stepped shaft
  - Crankshaft
  - Camshaft
  - Spline shaft



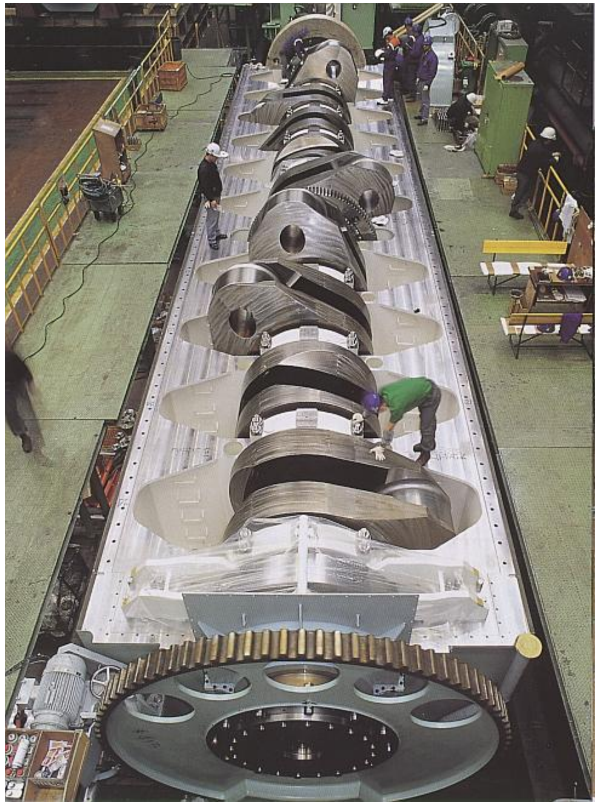


Camshaft

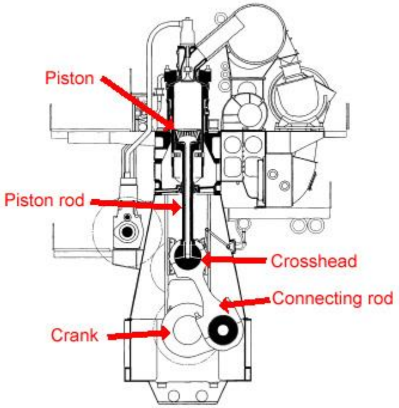


Spline shaft

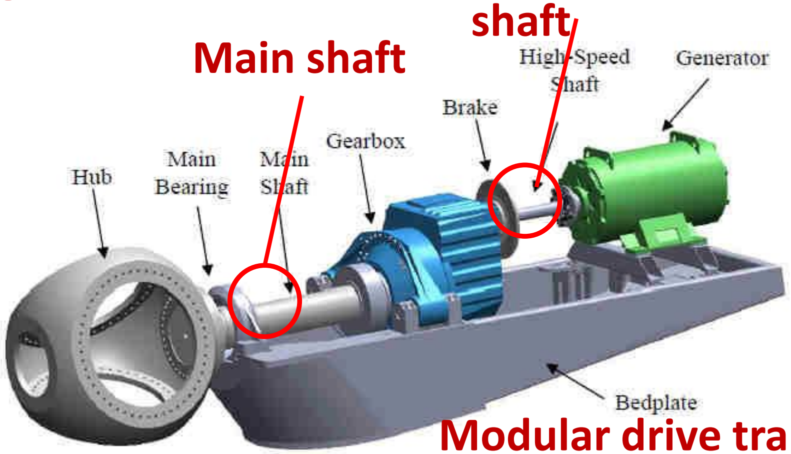
A few examples of shaft applications



Crankshaft of the most powerful diesel engine for container ships **(7.6x10^{6}Nm & 80MW, 26m & 300t)**
http://www.amusingplanet.com/2013/03/the-largest-and-most-powerful-diesel.html

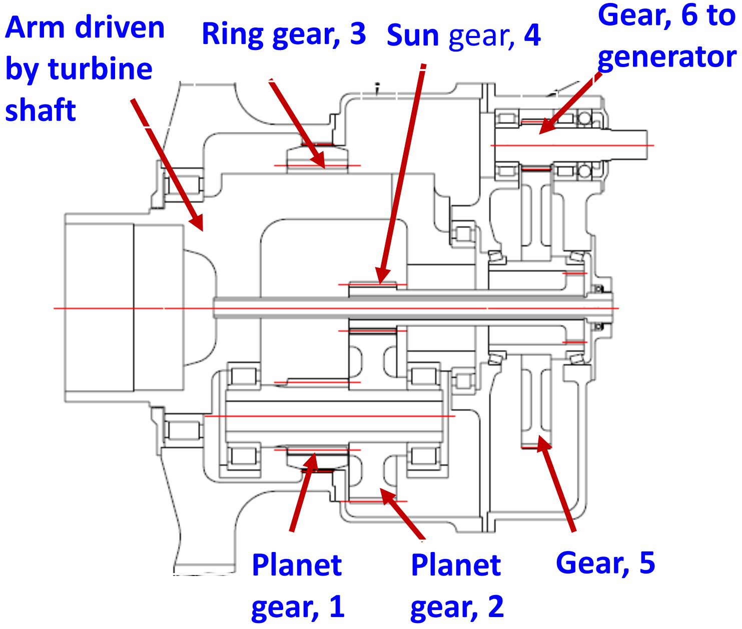


14-cylinder 2-stroke diesel engine



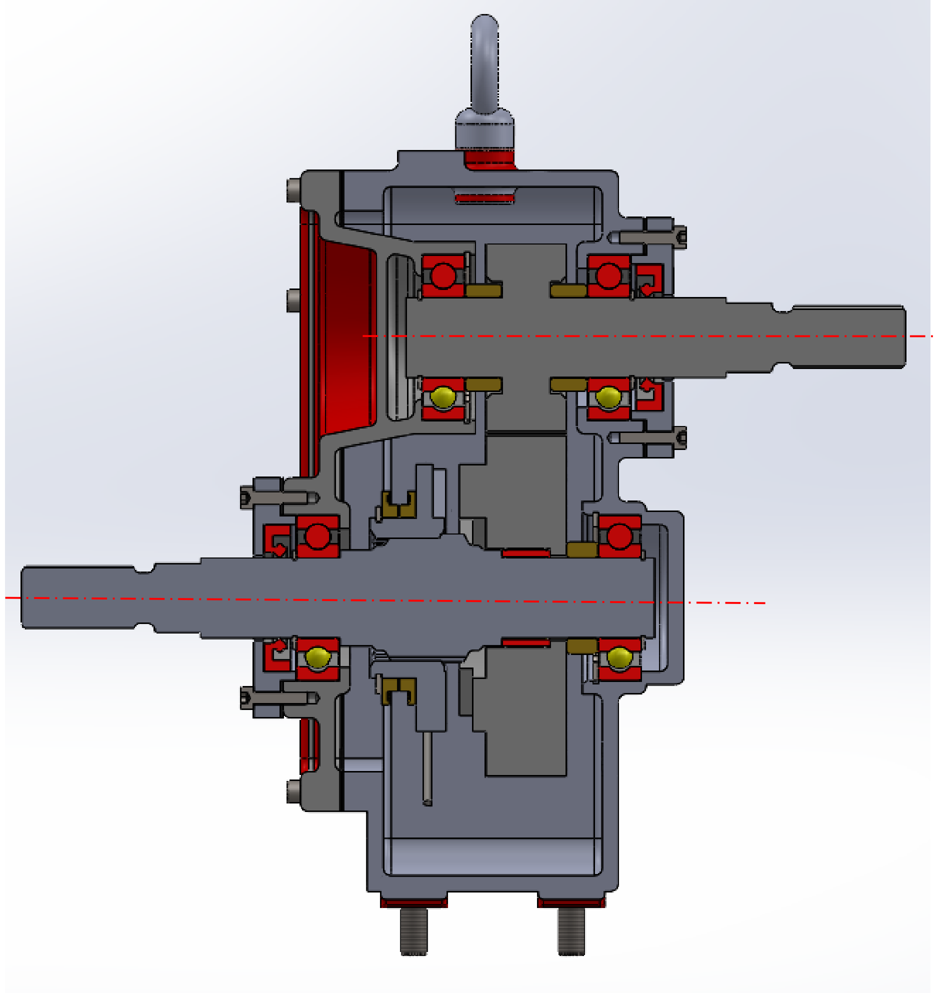
Modular drive train in Wind Turbine
http://www.nrel.gov/wind

Vestas **V90-3MW** wind turbine
http://www.vestas.com/

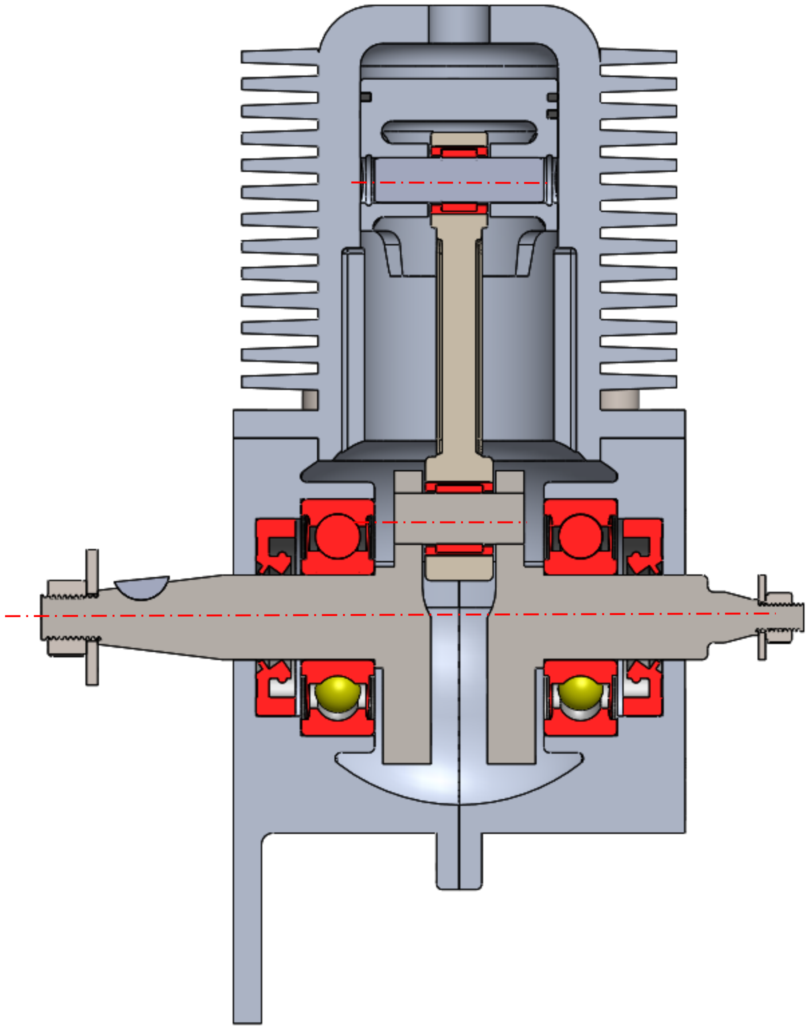


A **3MW** compound planetary helical gearbox,

A few more examples of shaft applications



Design of a Marine Gearbox

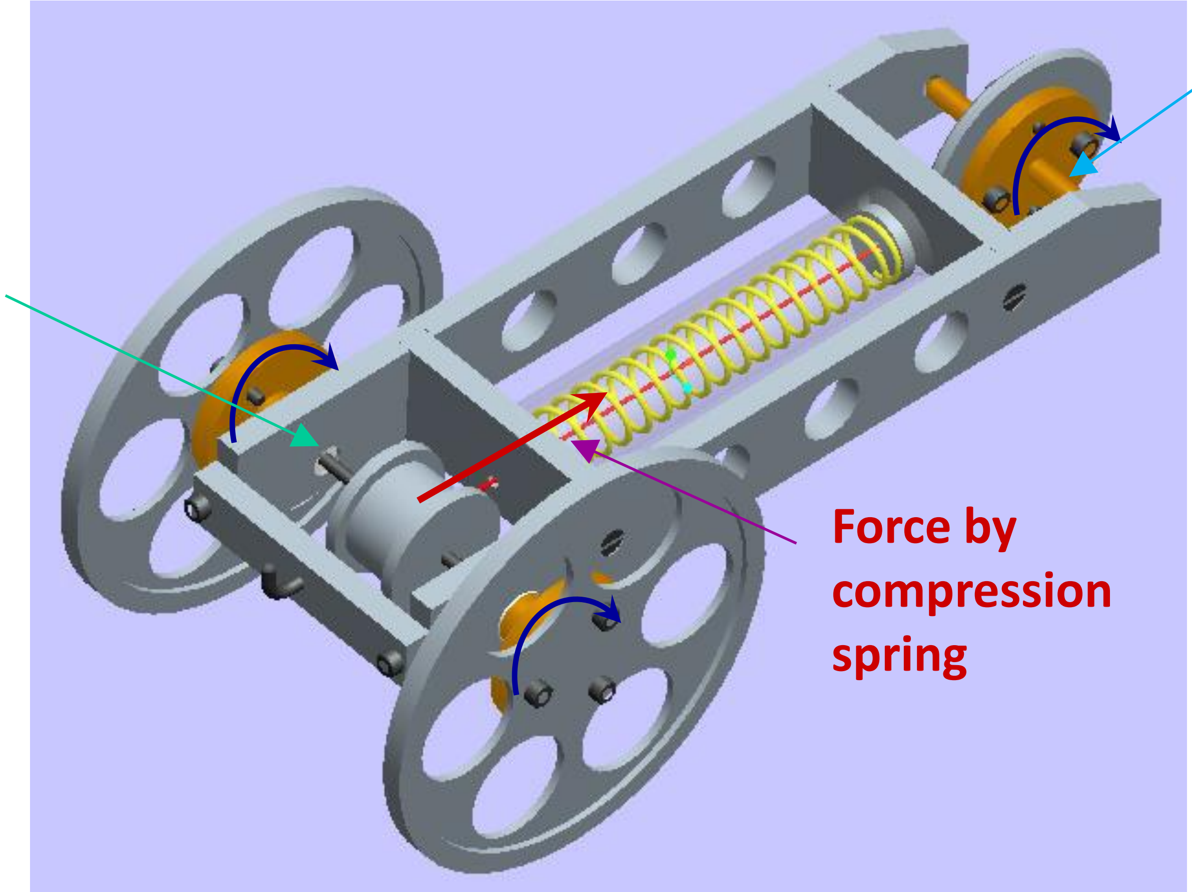


Design of a Two-Stroke Engine

Any more examples of shafts?

Rear axle (shaft) to

- apply torque for forward motion
- mount drum, bearings & wheels
- support chassis



Front axle (shaft) to

- mount front wheel
- support chassis

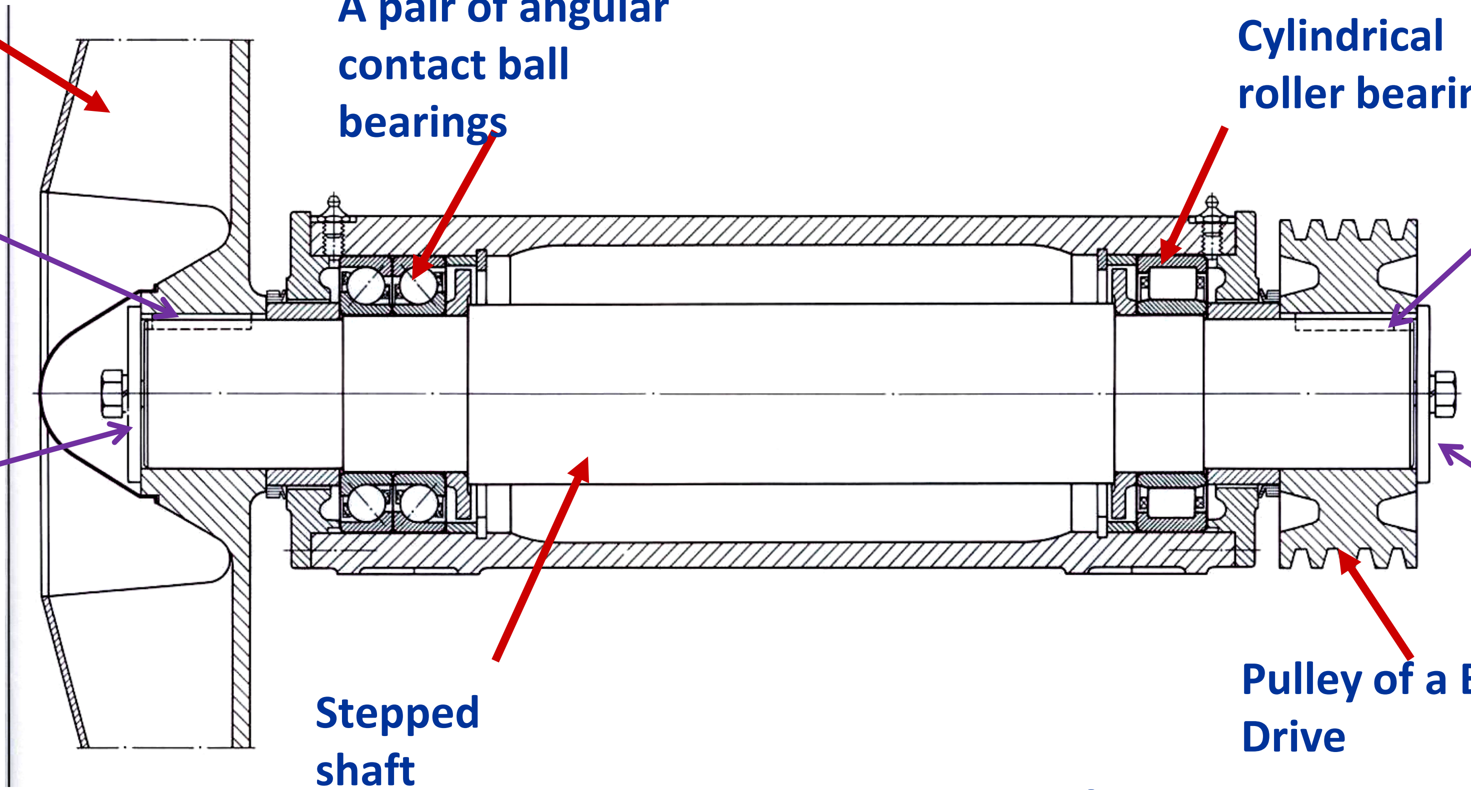
Force by compression spring

1st Year Group design & make project: Spring Powered Cart

Shaft design considerations

Shaft Design Part 1

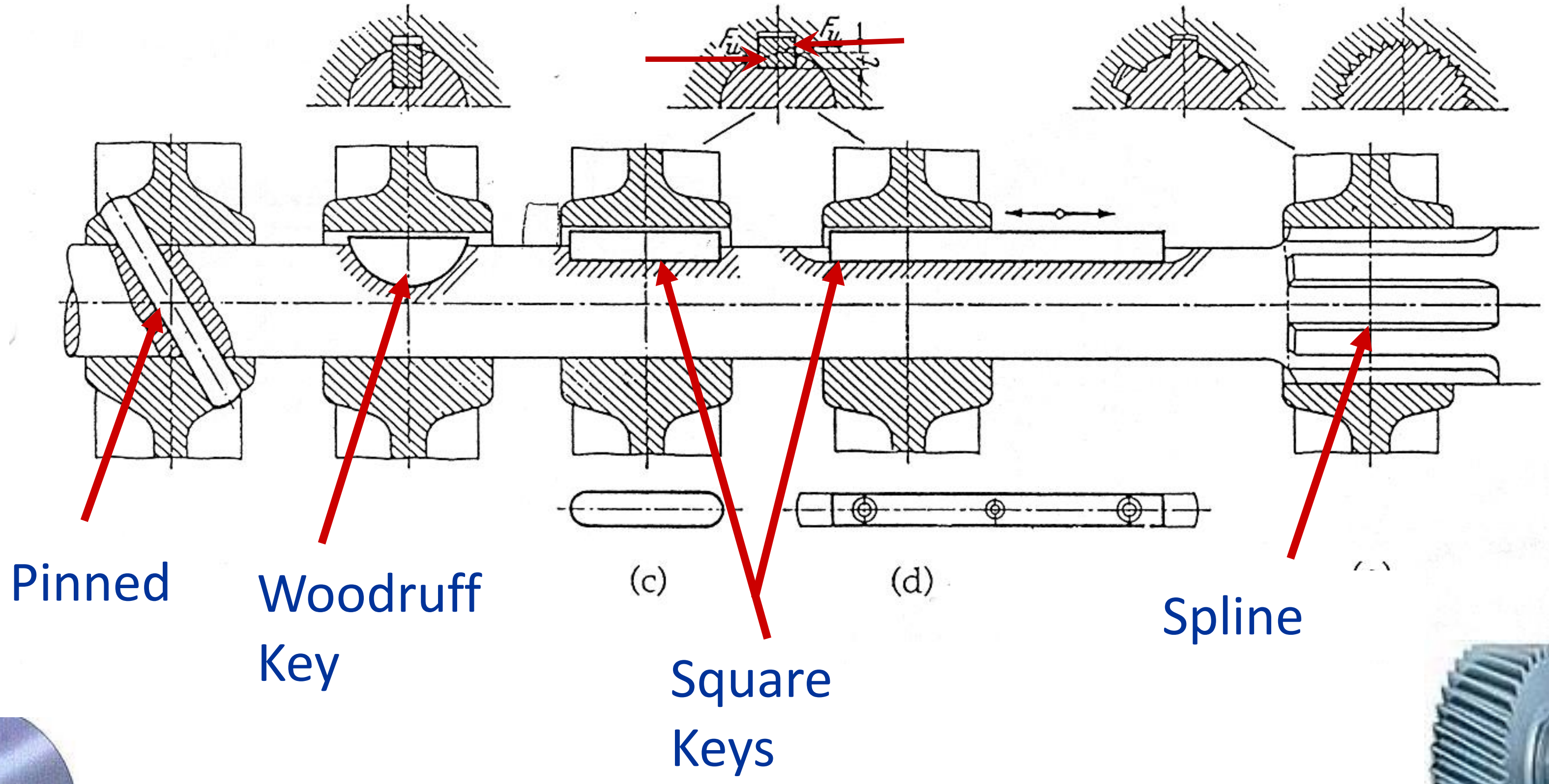
- Function & loading conditions
- Size & connection of components
- Material selection & treatments
- Deflection & rigidity
- Stress & strength
- Critical speed
- Manufacturing constraints
- Other design features

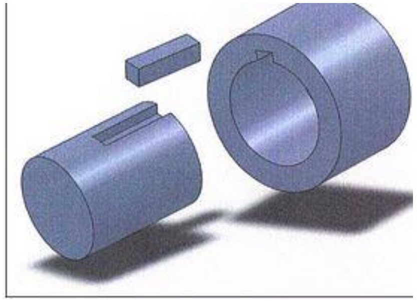


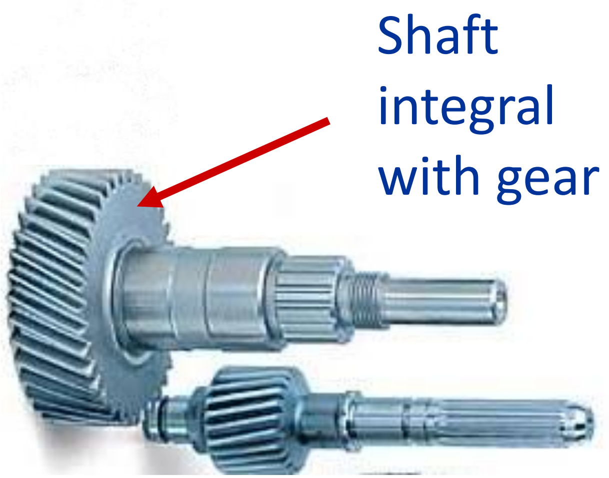
A fan supported by two bearings, SKF

Shaft-hub connections

- **Achieved by different types of keys and keyways and other means**

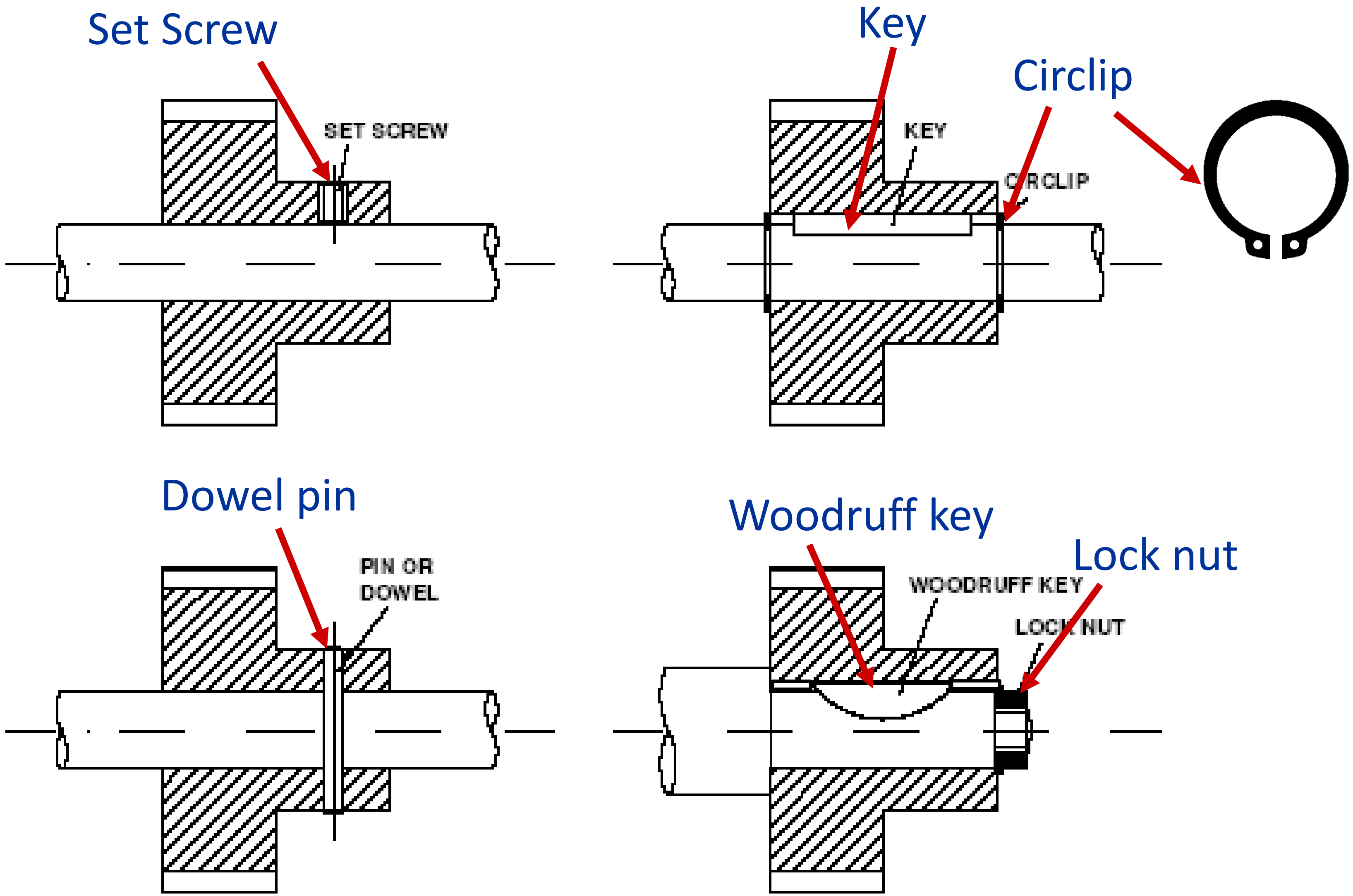






BS 4235-1:1972 (confirmed 2007) Specification for metric keys and keyways. Parallel and taper keys

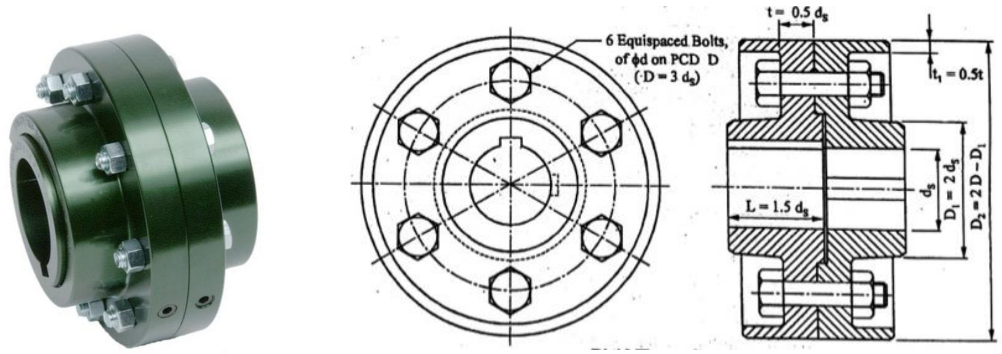
Shaft-hub connections



Shaft-shaft connections

(as covered from MMME1024)

- **Achieved by the use of rigid or flexible couplings**

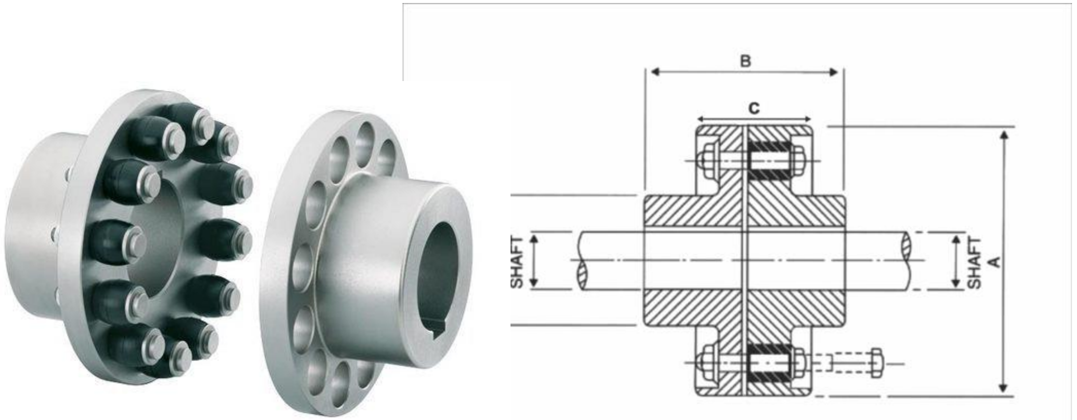


Rigid flange coupling

Hirth coupling

Clamp or split-muff coupling

Oldham coupling

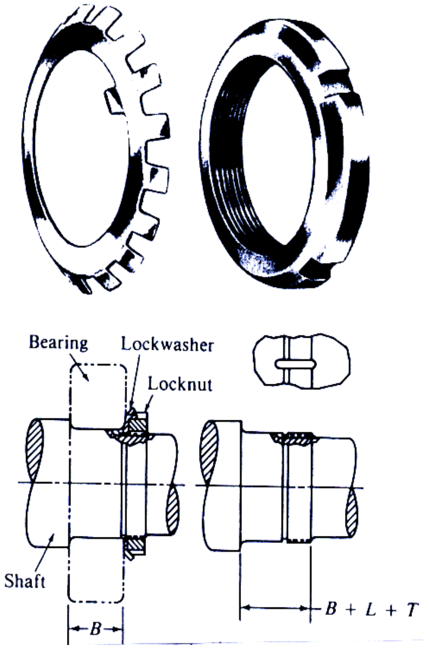


Bushed pin coupling

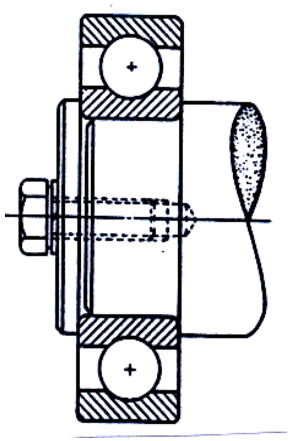
Gear coupling

Location of bearing on shaft

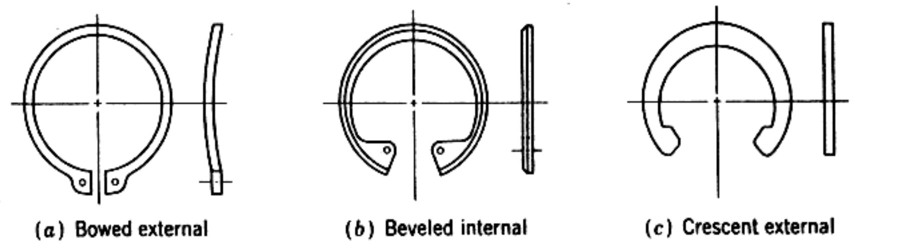
Means of locating outer/inner rings of bearings

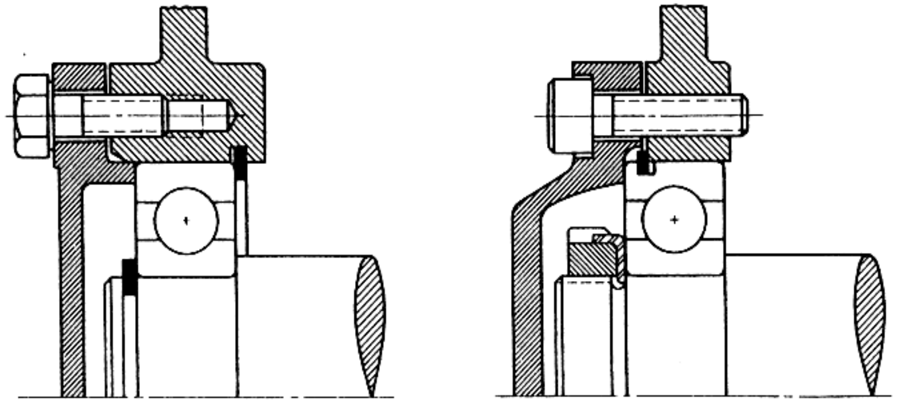


Lock nut &
lock washer



End plates



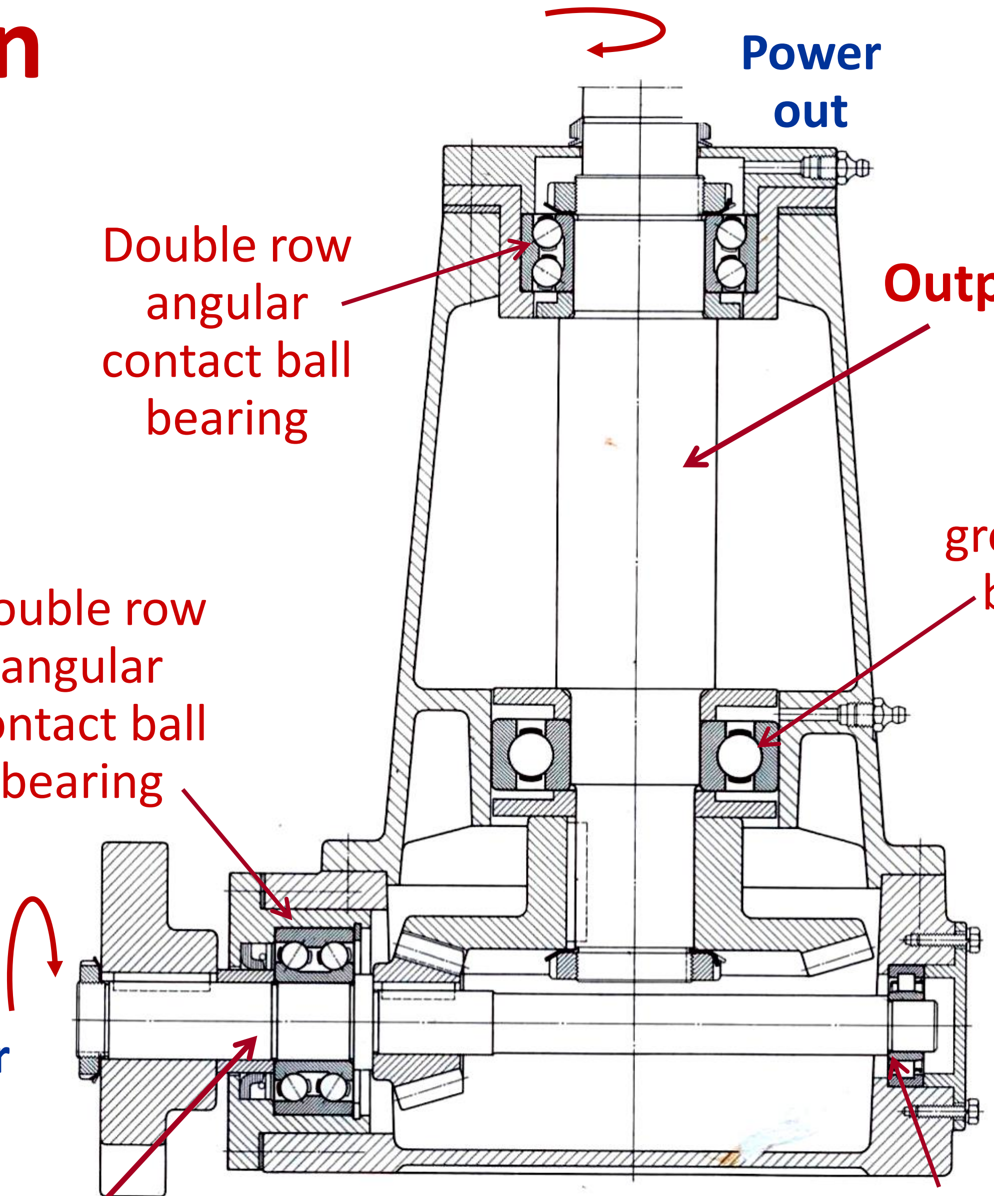


End cap, circlip & lock nut

An example of shaft-hub & shaft-shaft connection in design

- **A shaft** is normally supported, e.g. by rolling element **bearings** of various types
- At the same time, it is the **shaft** to provide the space for **bearing and gear assembly, etc**

Bevel gears



SKF Bevel gear unit

Location of bearing on shaft

Rotating ring: interference fit

Stationary ring: push (clearance) fit or transition fit

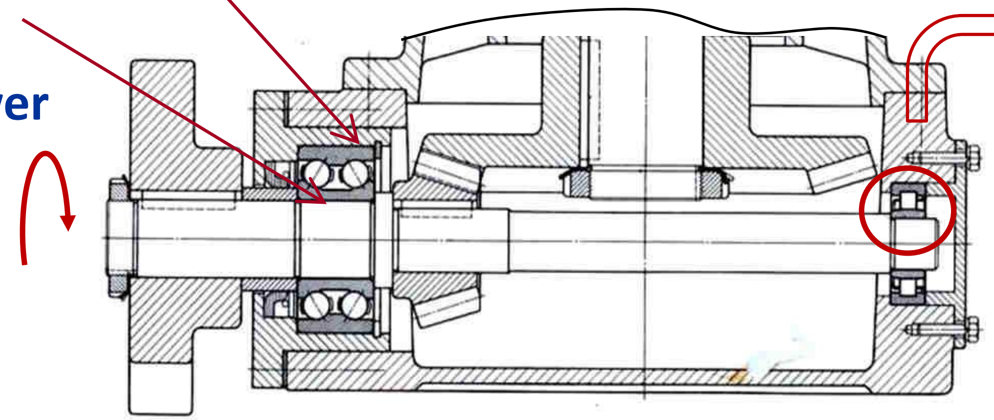
Axial location:

- Axial location of both rings against abutment faces

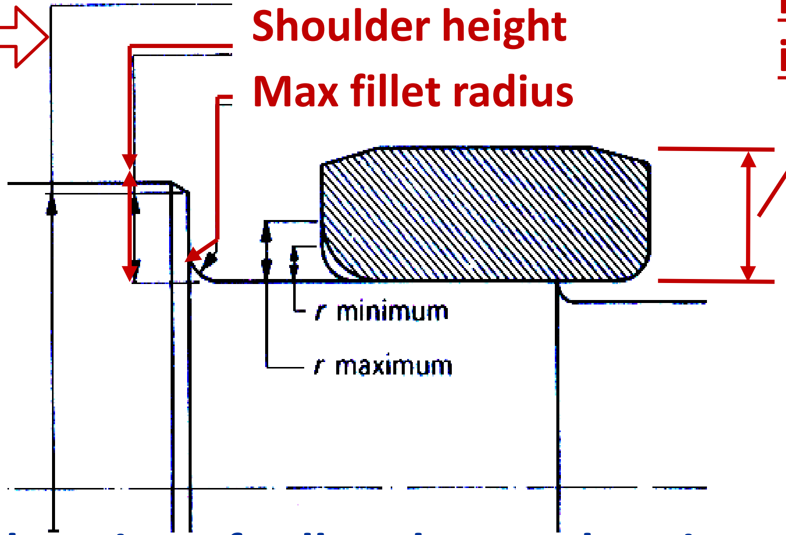
Shoulder fillet radius ≤ corner radius of bearing

Shoulder height ≈ 2~2.5 corner radius of bearing

< height of inner ring



A partial view of a SKF bevel gearbox



Axial location of roller element bearings

A quiz: Input shaft of a worm gearbox

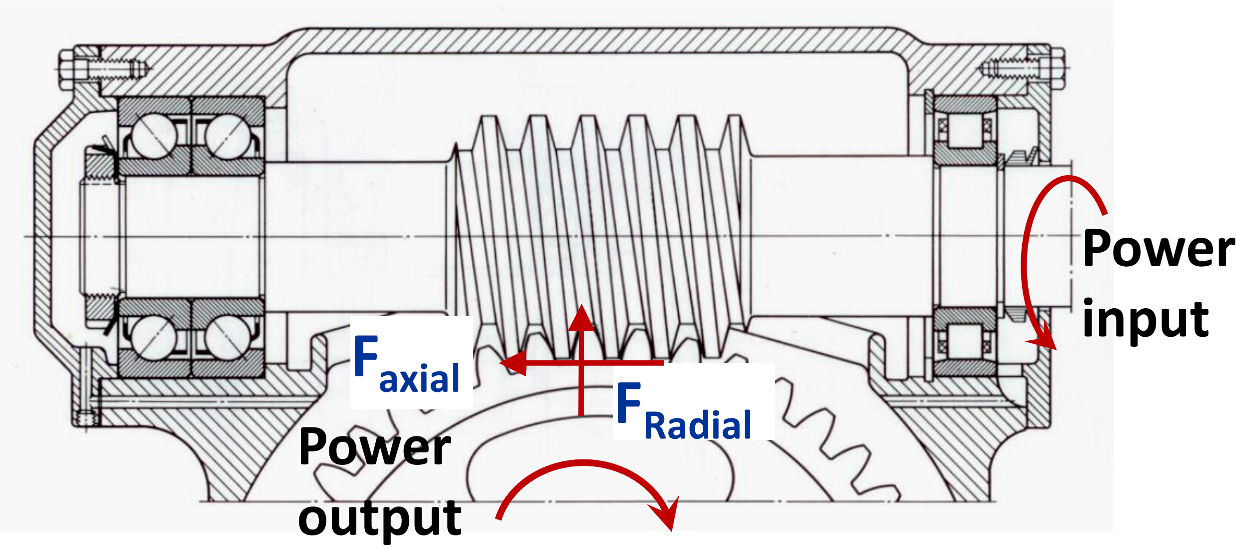
Which of following statements or design features are incorrect?

A. This input worm is made of a stepped shaft to provide bearing location in using lock nut and circlips.

B. A cylindrical roller bearing on the right and a pair of angular contact ball bearing on the left are used to support the input shaft.

C. A Clearance fit should be used for inner ring of cylindrical roller bearing.

D. The axial force (F_{axial}) at the meshing point is taken by the cylindrical roller bearing.



Statements C and D are incorrect because

C. An interference fit should be used

D. The axial force (F_{axial}) is taken by the pair of angular contact ball bearings

0% 0% 0% 0%

Shaft design

End of Part 1

Shaft design

Part 2

Shaft Design

Design procedure

1. Determine torque to be transmitted
2. Selection & position of bearings
3. Determine shaft loadings
4. Determine shaft speed

Iterative Process!

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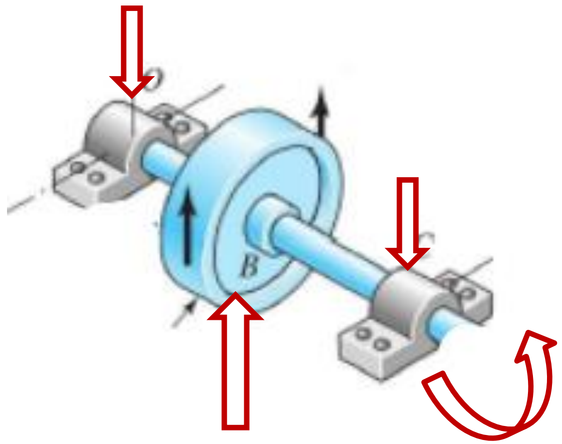
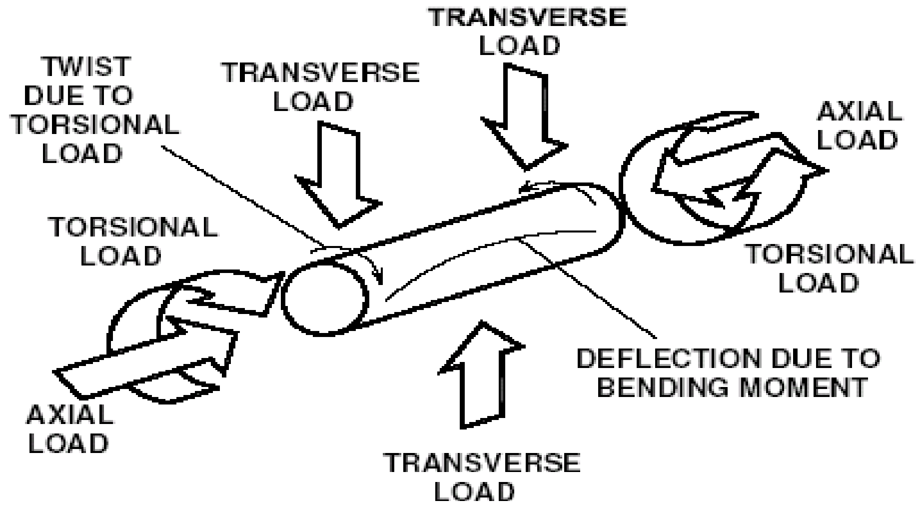
5. Determine shaft diameter – ASME Design Code
6. Design suitable design features
7. Apply Macaulay's method to determine shaft deflections
8. Apply Rayleigh-Ritz Equation to calculate **critical speed**

Shaft Loading

- **Axial stresses:**
  - due to self-weight in vertical shafts;
  - due to axial restraint at bearings and associated axial loads
- **Bending stresses:**
  - due to **bending moment in belt drives, gear forces**, mounted component weights (e.g. gear, flywheel)
  - dynamic forces which can lead to fatigue and resonance
- **Shear stresses:**
  - due to **torque load**/ direct shear



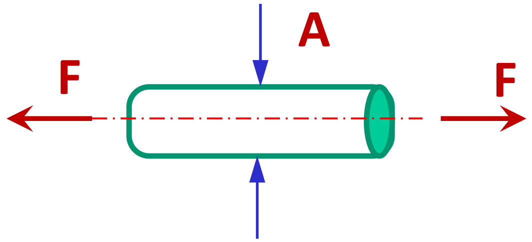
A pulley shaft sub-assembly



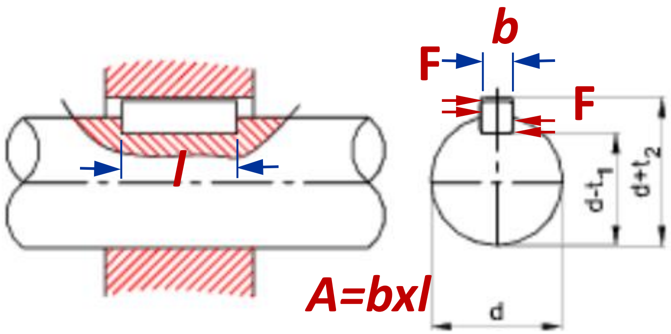
A general shaft loading case

Common types of loading and stresses

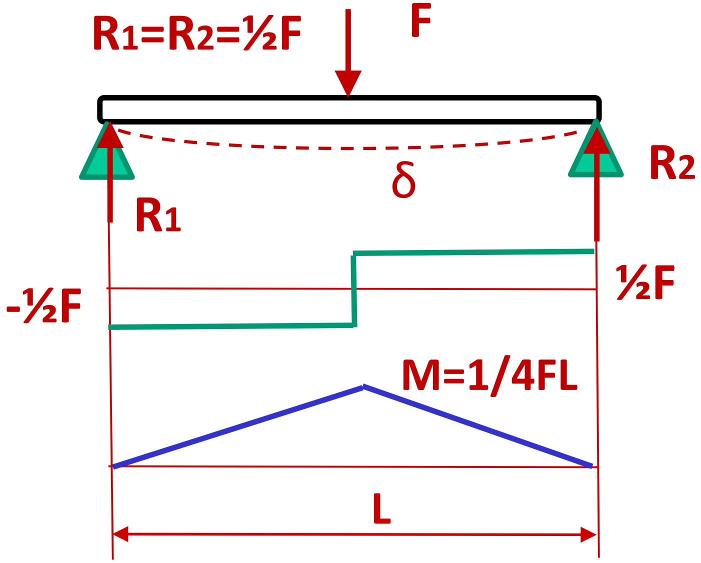
- **Types of loading & stress**
  - Tensile load $\sigma = \frac{F}{A}$
  - Direct shear $\tau = \frac{F}{A}$
  - **Bending** $\sigma = \frac{My}{I}$
  - **Torsion** $\tau = \frac{TR}{J}$



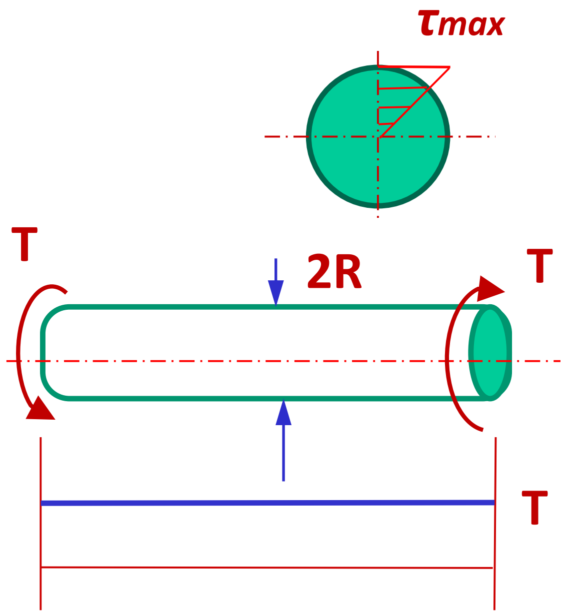
Tensile stress



Direct shear stress



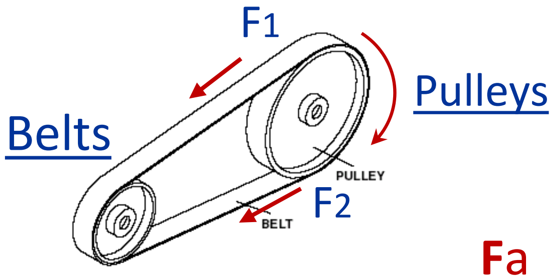
Bending moment & shear force

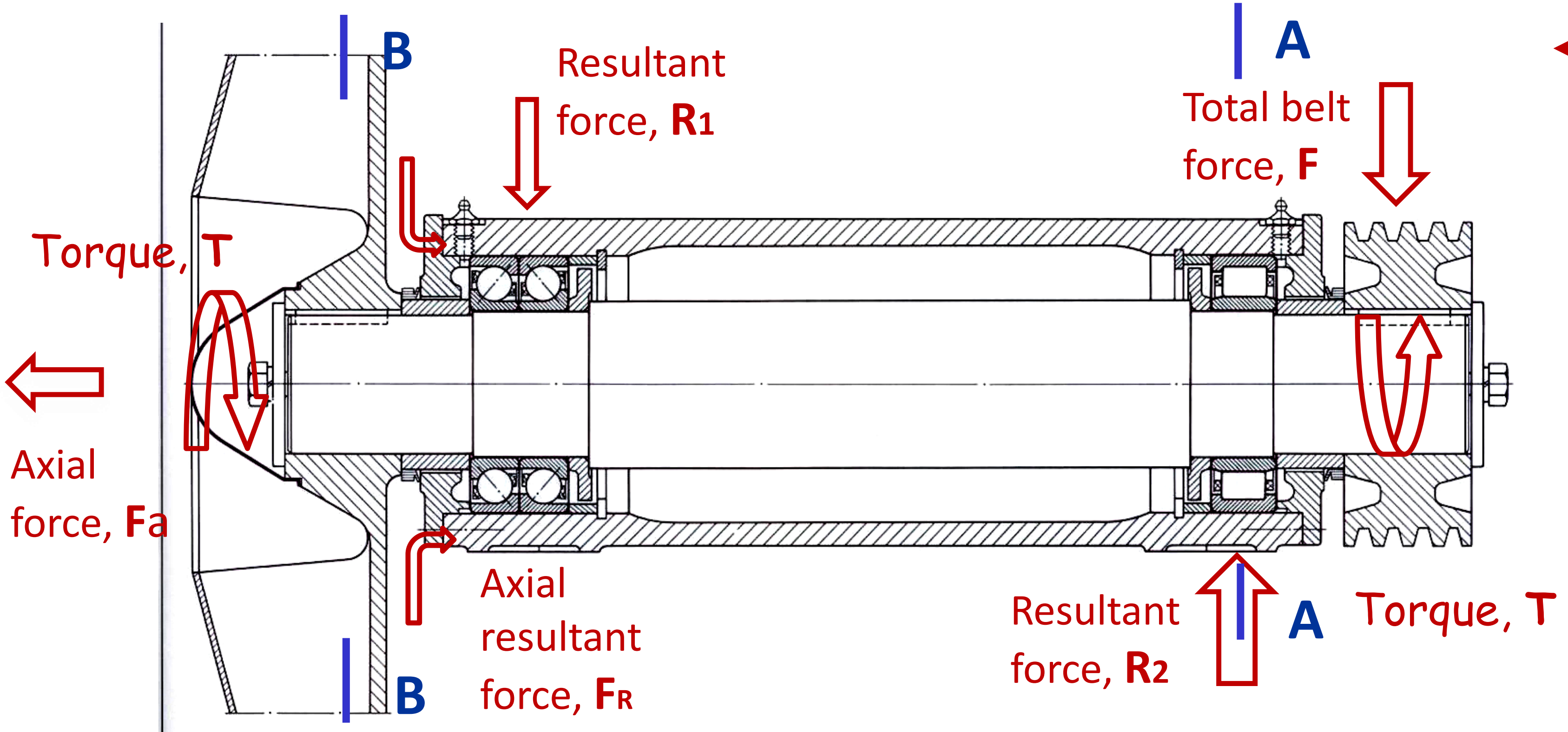


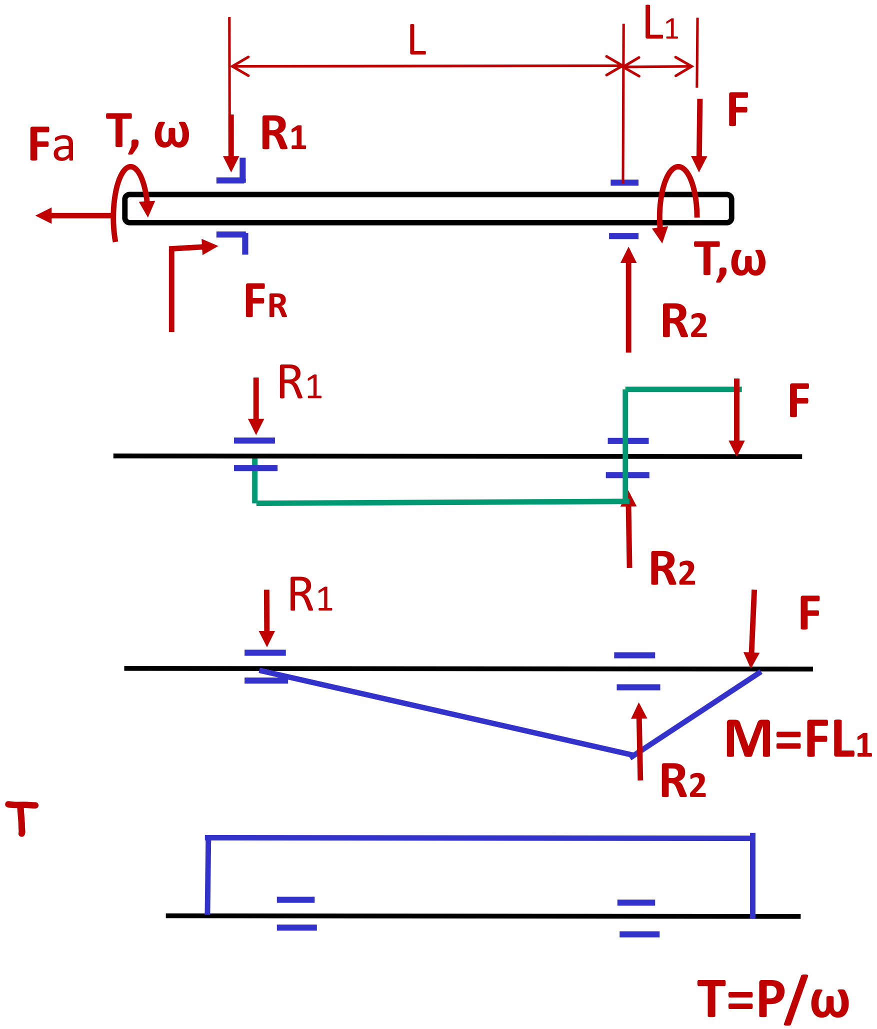
Shear due to torsion

An example of shaft loading

- **Concept of critical sections**







Sections A – A and B-B are critical sections

A SKF fan sub-assembly

Beam bending shear force, moment and torque diagrams (MMME1028)

Shaft diameter

Use the ASME design code to calculate shaft diameter

Minimum diameter of shaft: *dmin* (m)

Note: ASME (American Society of Mechanical Engineers)

$$d = \left[\frac{32 n_s}{\pi} \sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2} \right]^{1/3}$$

- n_s: **Reserve or safety factor (often use 2)**
- T: Max torque on shaft (**Nm**)
- σ_y: Yield strength of shaft material (**Pa**)
- M: Max bending moment on shaft (**Nm**)
- σ_e: Endurance limit stress (**Pa**)

Note: be careful about the units in calculation

A guide for reserve or safety factor

Reserve (or safety) **factor** is a simple and robust approach to accommodate **uncertainties** in engineering design and evaluation.

Recommended reserve factor (n_s)	Operational and environmental conditions as well as use of materials	Uncertainty of material and working conditions
1.25 ~ 1.5	Reliable materials under controlled conditions subjected to loads and stresses known with certainty	
1.5 ~ 2	Well-known materials under reasonably constant environmental conditions subjected to known loads and stresses	
2 ~ 2.5	Average materials subjected to known loads and stresses	
2.5 ~3.0	Less well-known materials under average conditions of load, stress, and environment	
3 ~ 3.4	Untried materials under average conditions of load, stress, and environment or well-known materials under uncertain conditions of load, stress, and environment	

Shaft diameter

How is the ASME design code equation derived?

- **Bending moment** creates **alternating tensile/compressive stresses** (σ_a)

$$\sigma = \frac{My}{I} \qquad I = \frac{\pi d^4}{64} \quad \Rightarrow \quad \boldsymbol{\sigma_a = \frac{32M}{\pi d^3}}$$

- **Torque** normally generates **constant shear stress** (τ_m)

$$\tau = \frac{TR}{J} \qquad J = \frac{\pi d^4}{32} \quad \Rightarrow \quad \boldsymbol{\tau_m = \frac{16T}{\pi d^3}}$$

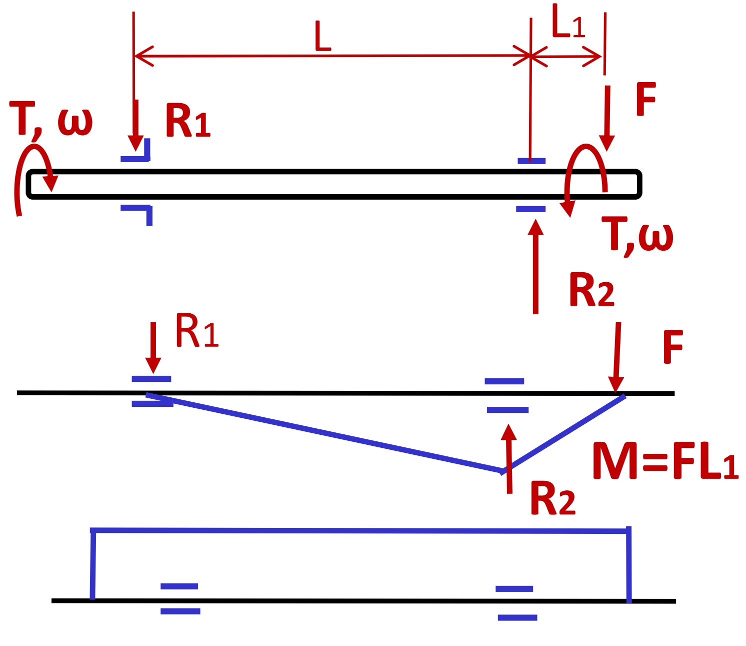
- von Mises stress in **plane stress** condition

$$\boldsymbol{\sigma_{vm} = \left(\sigma^2 + 3\tau^2\right)^{1/2}}$$

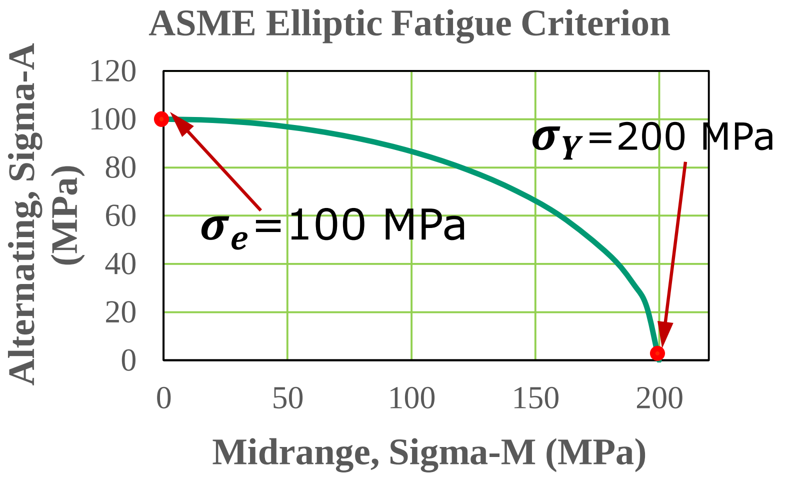
- **ASME Fatigue Failure Criterion**

$$\left(\frac{n_s\sigma_a}{\sigma_e}\right)^2 + \left(\frac{n_s\sigma_m}{\sigma_Y}\right)^2 = 1 \quad \Rightarrow \quad d = \left[\frac{32n_s}{\pi}\sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2}\,\right]^{1/3}$$

Note: von Mises alternating (σ_a) and midrange (σ_m) stresses



Use SKF fan assembly as example



Shigley's Mechanical Engineering Design, R.G. Budynas & J.K. Nisbett, 10th ed, 2015, McGraw Hill

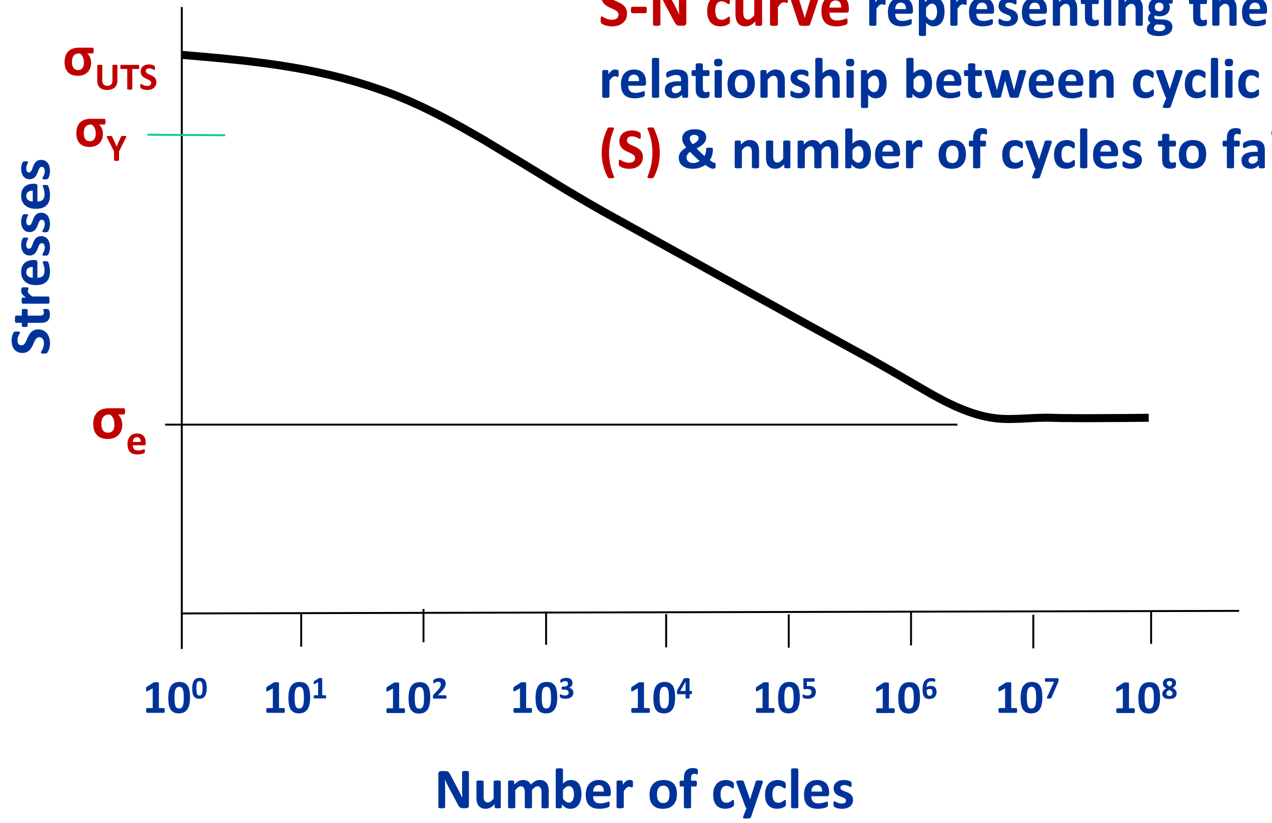
Shaft diameter

Endurance limit stress (σ_e) is a limit level of stress, with which fatigue failure wouldn't occur in under cyclic or alternating load condition.

σ at failure decreases for increased cyclic loading

After more than 10^6 cycles, reduction in σ stops.

Material will last for **"infinite"** cycles so long as the endurance limit stress, σ_e, isn't exceeded



S-N curve representing the relationship between cyclic stresses (S) & number of cycles to failure (N)

Most steels have this fatigue behaviour -> Often used for shafts

Shaft diameter

Endurance limit stress, σ_e, is affected by factors such as loading, reliability and stress concentration of a specific shaft design, etc

$$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma_e^{'}$$

$\sigma_e^{'}$ **- Endurance limit of test specimen**

Where,

k_a = surface factor

k_b = size factor (=1)

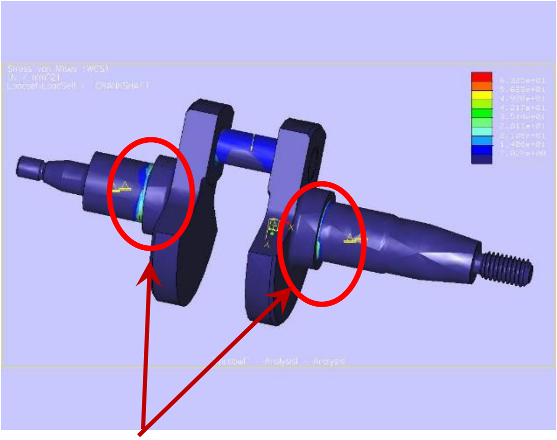
k_c = reliability factor

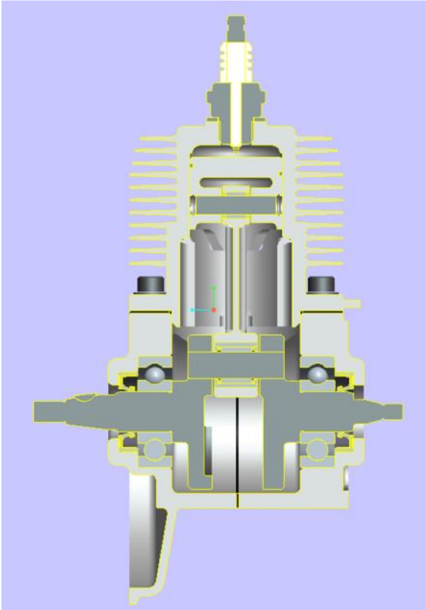
k_d = temperature factor (=1)

k_e = duty cycle factor (=1)

k_f = **fatigue stress concentration factor**

k_g = miscellaneous effects factor (=1)





Stress concentration in crankshaft of a 2-stroke engine

Check Handouts for specific values of all the factors

Shaft diameter

$$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma_e^{'}$$

Endurance limit stress of test specimen (σ'_e) is related to σ_{UTS}

$$\sigma_e^{'} = 0.504\sigma_{UTS} \quad for\ \sigma_{UTS} \leq 1400\,MPa$$

$$\sigma_e^{'} = 700\,MPa \quad for\ \sigma_{UTS} \geq 1400\,MPa$$

where,

k_a = surface factor

k_b =1

k_c = reliability factor

k_d = 1

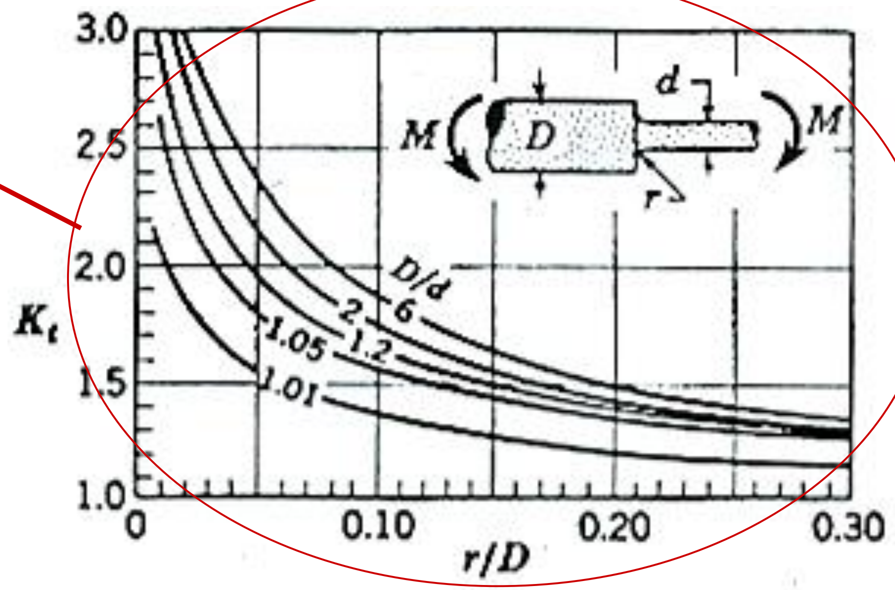
k_e = 1

k_f=stress concentration factor

k_g = 1

$$k_f = \frac{1}{1 + q(K_t - 1)}$$

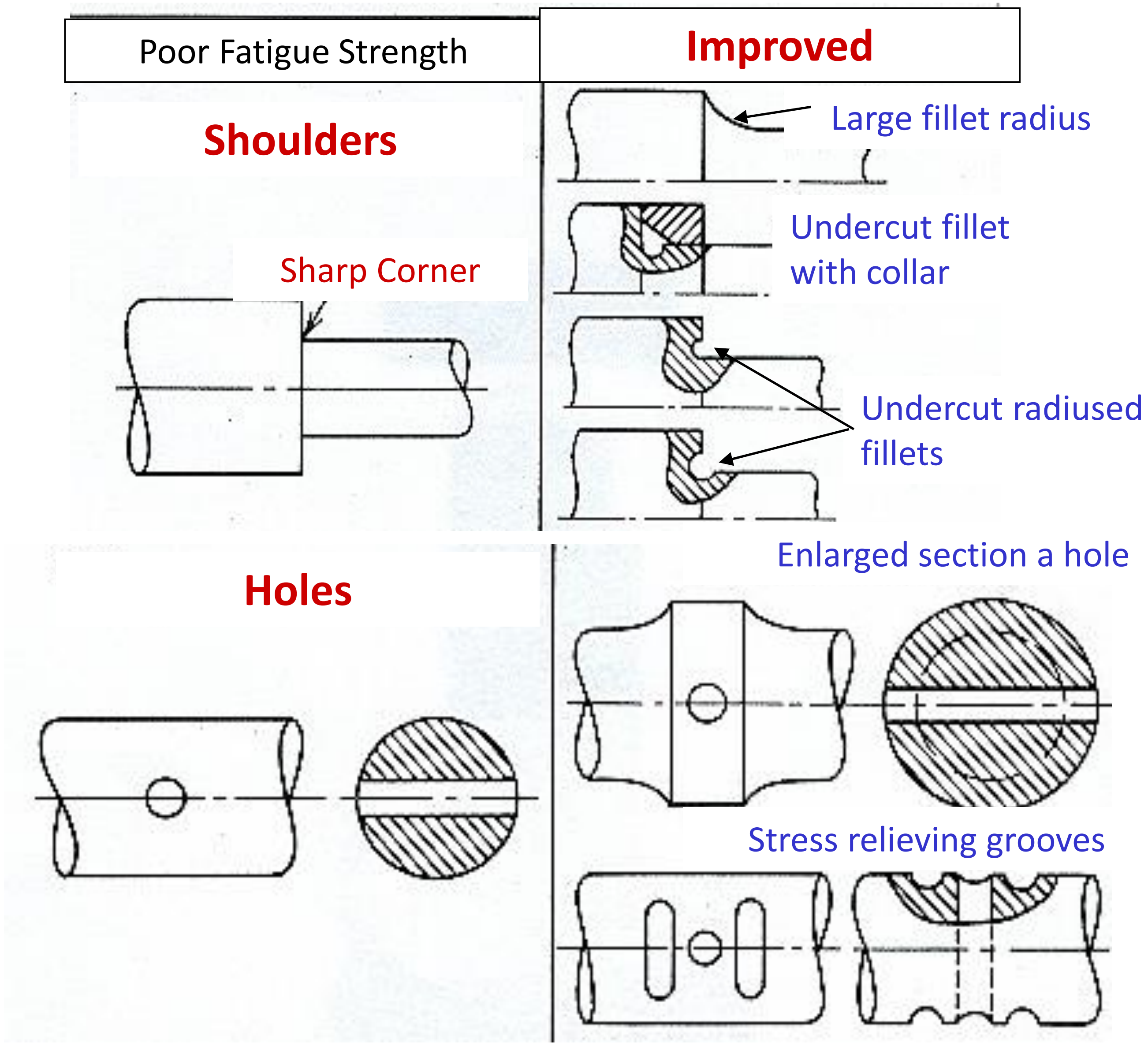
q is notch sensitivity factor (*Fig. 4 of handout*) and ***Kt*** is geometric stress concentration factor (*Fig. 5 of Handout*)



Select all ***k* factors** from graphs, tables, empirical formulas as **given in Shaft Design handouts**

Shaft fatigue

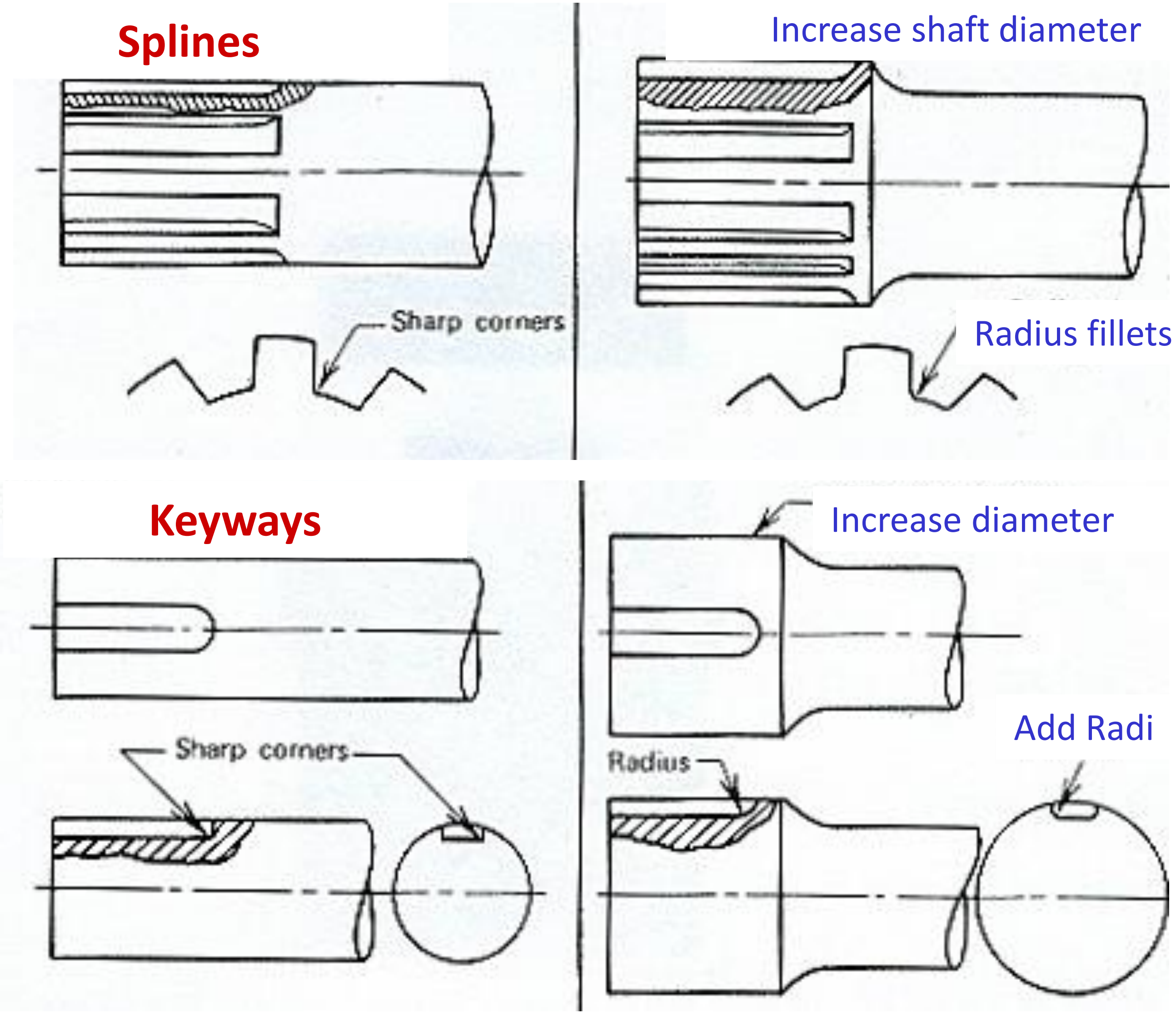
"Best practice" to minimise **stress concentration** -> fatigue



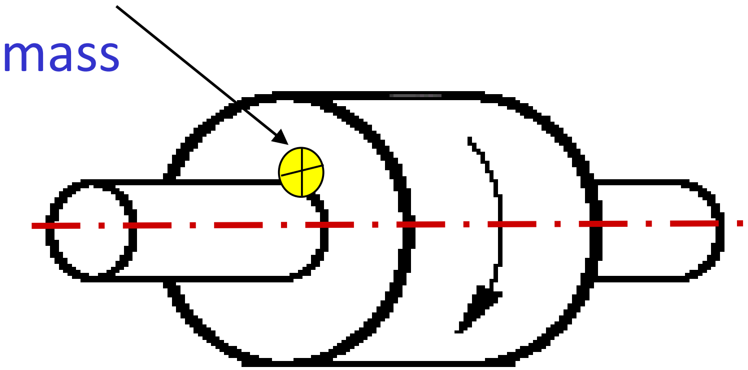
Shaft fatigue

"Best practice" to minimise stress concentration -> fatigue

Poor Fatigue Strength	Improved



Critical speed of shaft



Centre of Mass should equal Centre of Rotation

(but in practice it doesn't)

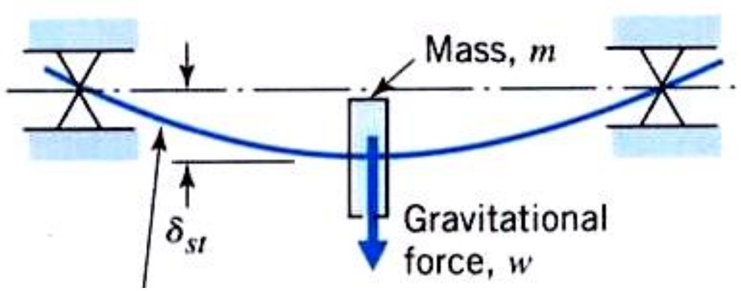
Imbalance causes a deflection

(**centrifugal force**, **$mr\omega^2$** normally balanced by flexural rigidity, EI)

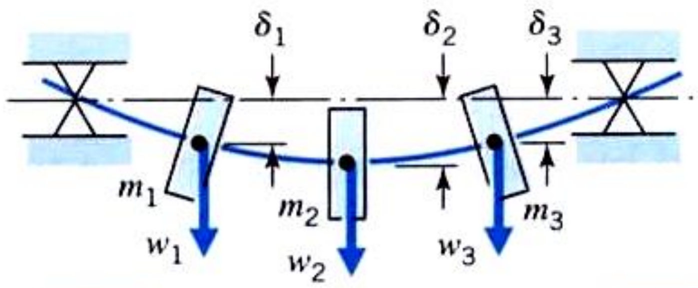
At the **critical speed** (**natural frequency**) shaft is unstable

(deflection increases significantly to break shaft, damage bearing and cause destructive vibration, "**shaft whirling**")

Critical speed of shaft



Shaft with a single mass



Shaft with multiple masses

First Critical Speed

$$\omega_C = \sqrt{\frac{g}{\delta_{st}}} \quad \left(rad/s\right)$$

where,

g – acceleration of gravity (m/s^2),

$\boldsymbol{\delta_{st}}$- static deflection of shaft (m).

Rayleigh-Ritz equation

$$\omega_C = \sqrt{g\frac{\sum w_i \delta_i}{\sum w_i \delta_i^2}} \left(rad/s\right)$$

where,

$\boldsymbol{w_i}$ – weight of node **i** (N),

$\boldsymbol{\delta_i}$ - static deflection of node **i** (m).

Rule of Thumb

Operational speed of shaft should be ½ the critical speed

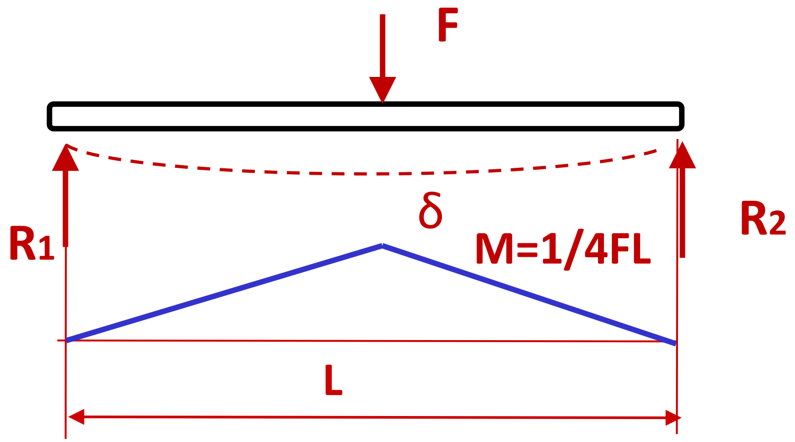
Shaft deflection

Shaft deflection is required to determine **critical speed**

- Macaulay's method for the deflection in Beam Bending (**MMME2053**):

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad y = \int_0^x \int_0^x \left(\frac{M}{EI} dx \right) dx + C_1 x + C_2$$

As an example, a plain shaft (a case of simply supported beam) under a force **F** at the centre of shaft, the maximum deflection is



$$\delta_{max} = \frac{FL^3}{48EI}$$

where,

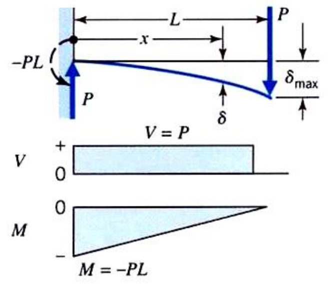
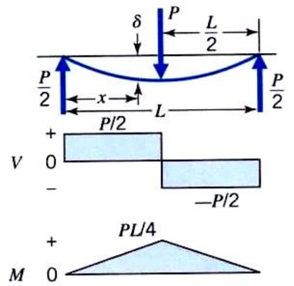
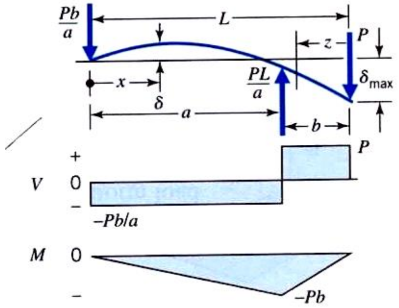
F – force applied (N), ***E*** – Young's modulus (N/m^2)

L – span of the beam (m), ***I*** – 2nd moment of area (m^4)

Note: be careful about the units in calculation

Shaft deflection

Shaft Design Part 2

<table>
<tr><th>Types of beams</th><th>Max deflection</th><th>Deflection at any point x</th></tr>
<tr><td>
</td><td>$$\delta_{max} = \frac{PL^3}{3EI}$$</td><td>$$\delta = \frac{Px^3}{6EI}(3L - x)$$</td></tr>
<tr><td>
</td><td>$$\delta_{max} = \frac{PL^3}{48EI}$$</td><td>$$\delta = \frac{Px}{12EI}\left(\frac{3L^2}{4} - x^2\right)$$</td></tr>
<tr><td>
</td><td>$$\delta_{max} = \frac{Pb^2L}{3EI}$$</td><td>$For\ 0 \le x \le a:$
$$\delta = \frac{Pbx}{6aEI}(x^2 - a^2)$$
$For\ 0 \le z \le b:$
$$\delta = \frac{Pbx}{6EI}(z^3 - b(2L+b)z + 2b^2L)$$</td></tr>
</table>

Summary

- To be familiar with shaft function, types and applications
- To be able to design appropriate shaft-hub and shaft-shaft connections
- To be able analyse **shaft loading**
- To be able to determine **shaft diameter** using ASME design code
- To design features for **preventing fatigue**
- To determine **shaft deflection** & **critical speed**

Revision Questions

a) Name a few common types of shafts.

b) What are the key consideration factors in designing a shaft?

c) What are the common means used in shaft-hub and shaft-shaft connections?

d) What would be the suitable fits of the inner ring of a rolling bearing to support a rotating shaft and the outer ring mounted in housing?

e) How would you define a critical cross-section when you calculate the minimum shaft diameter using ASME design code?

f) What is Endurance Limit Stress (σ_e) and how is it affected in shaft design?

g) What are the common features that would create stress concentration in shaft design?

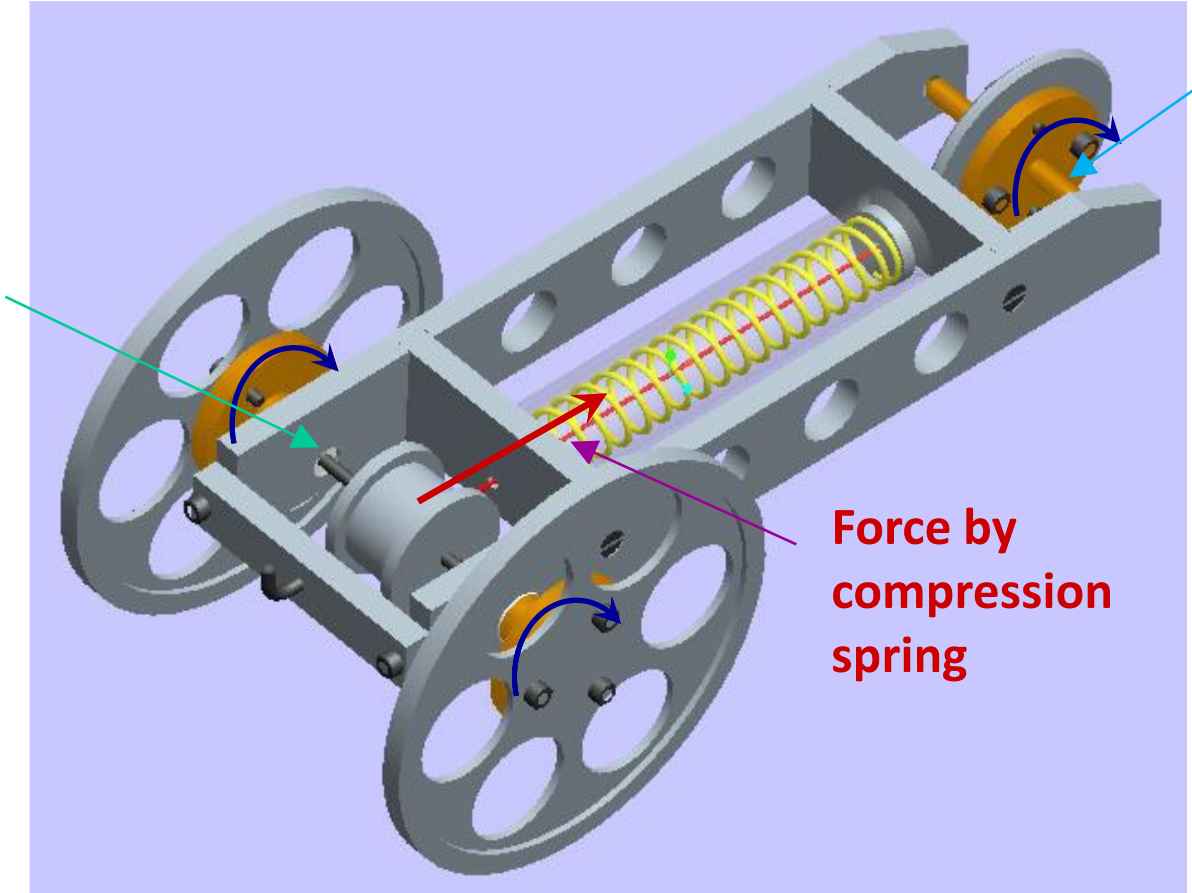
h) What is the rule of thumb to avoid "shaft whirling"?

Worked example 1

- Design the drive shaft of the spring powered cart

Rear axle (shaft) to

- apply torque for forward motion
- mount drum, bearings & wheels
- support chassis



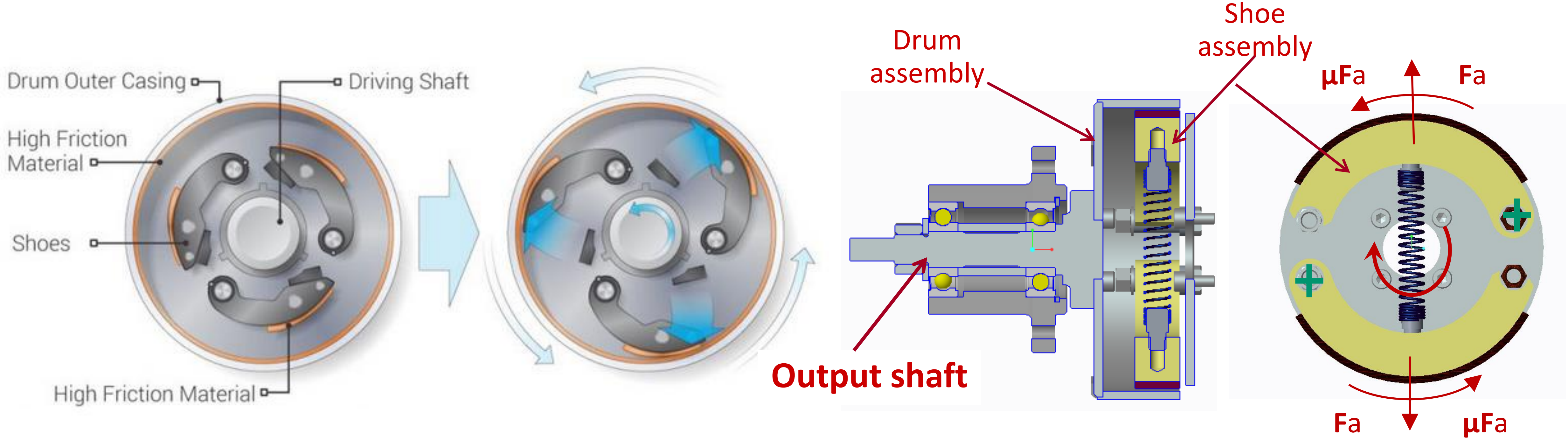
Front axle (shaft) to

- mount front wheel
- support chassis

1st Year Group design & make project: Spring Powered Cart

Worked example 2

- Design the output shaft of the centrifugal clutch



What is a centrifugal clutch?

A mechanical device to allow smooth connection of two rotating shafts by centrifugal forces
(more details will be covered in **Brakes & Clutches Lecture**)

https://www.youtube.com/watch?v=v6opn_jZMAE

https://www.youtube.com/watch?v=KL_P1mNMx_k

Worked example 2

- Design the output shaft of the centrifugal clutch

Material:
Bright Drawn Mild Steel (BDMS)
σ_{UTS} = 320 MPa & σ_y = 220 MPa.
Reserve factor, n_s=2

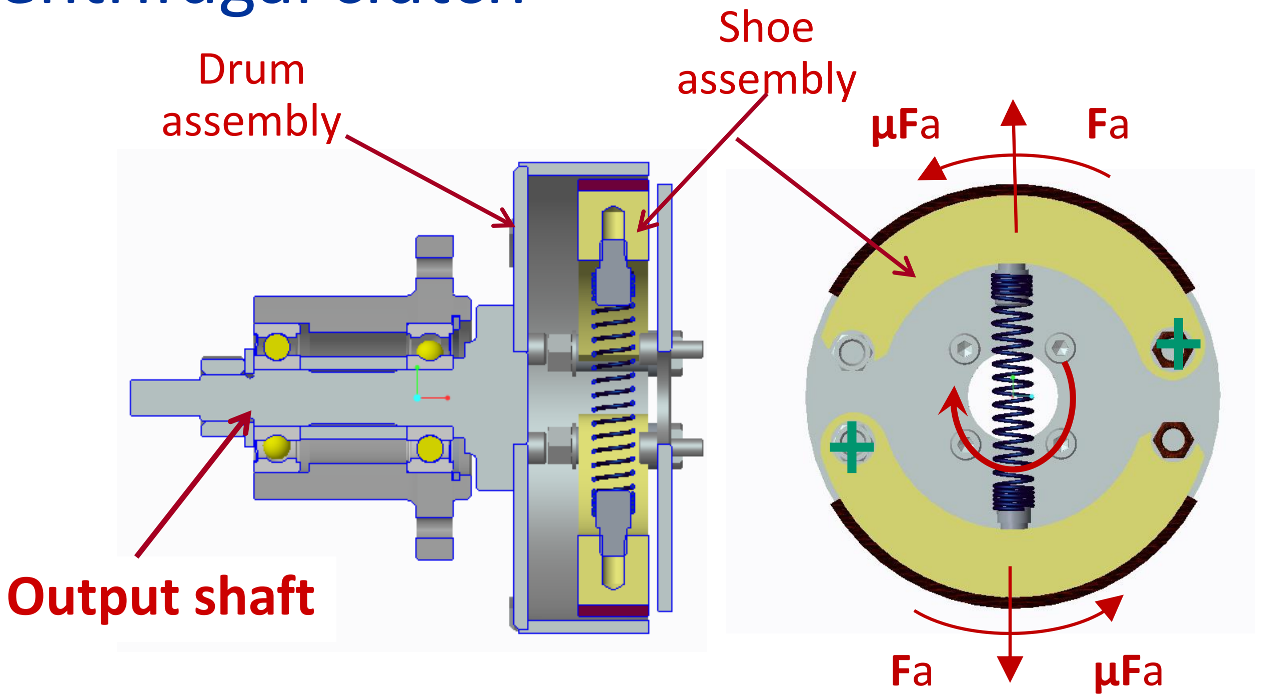
Maximum torque 0.5 Nm at 2300 rpm

Design the output shaft

What is the loading condition of the output shaft?

How to determine the minimum diameter?

How to design necessary features for shaft connection?



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Exercise two solution

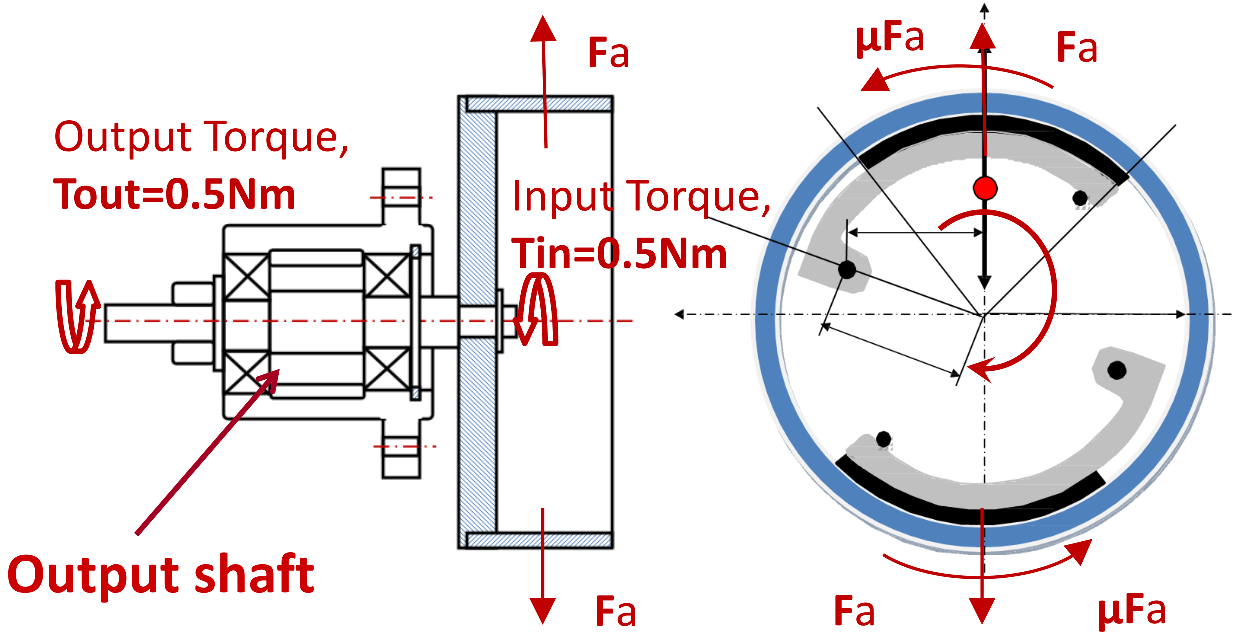
- Designing the output shaft of centrifugal clutch

Material:
Bright Drawn Mild Steel (BDMS)
σ_{UTS} = 320 MPa & σ_y = 220 MPa.
Reserve factor, **ns**=2

Maximum torque is 0.5Nm

Shaft loading conditions

Minimum diameter of the shaft



$$d = \left[\frac{32n_s}{\pi}\sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2}\right]^{1/3} = \left[\frac{32\times 2}{3.1416}\sqrt{\left(\frac{0.0}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{0.5}{220\times 10^6}\right)^2}\right]^{1/3} = 3.423\times 10^{-3}(m)$$

Although the minimum shaft dia is **3.5** mm, in practical design a larger diameter may be used for practical considerations.

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Shaft design

End of Part 2

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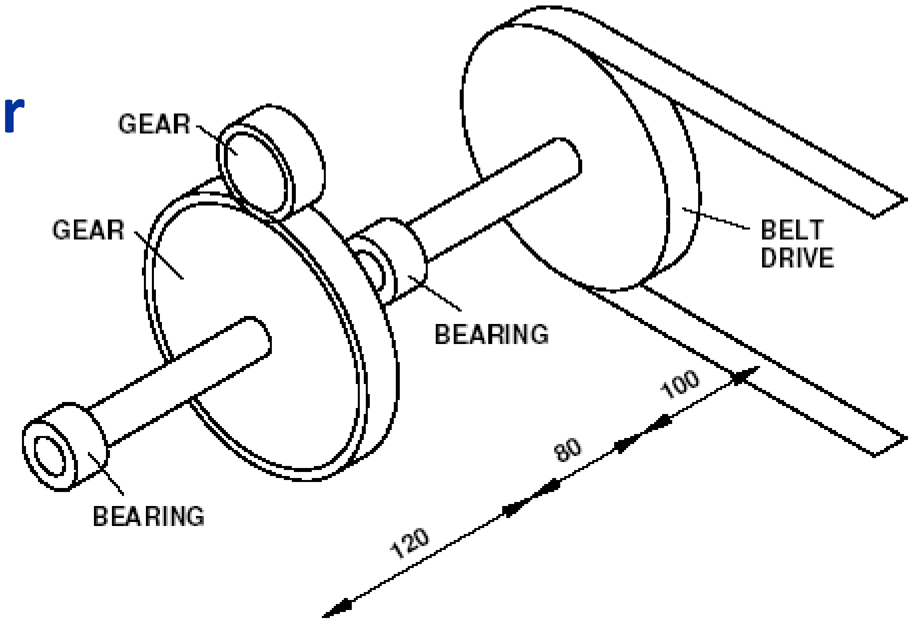
Shaft design

Part 3 Worked examples

An additional worked example

- Determining Shaft diameter of a transmission shaft with belt & spur gear drives

Calculate the minimum shaft diameter



Material:

817M40 hot rolled alloy steel

σUTS = 1000 MPa

σy = 770 MPa.

Brinell hardness is approximately 220 BHN.

Reserve factor, **ns** = 2

Features:

Radii on fillets is **r** = 3 mm.

Reliability required is 90%

Output:

8 kW at 900 RPM with a **Torque of ??**

$$P_{(W)} = T_{(Nm)}\omega_{(1/s)}$$

$$\omega_{(1/s)} = \frac{2\pi}{60} n_{(rpm)}$$

$$T_{(Nm)} = \frac{60\times10^3 P_{(kW)}}{2\pi\, n_{(rpm)}}$$

See Handouts for detailed calculations

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Shaft design

End of Part 3